HIGH-TEMPERATURE DRILLING

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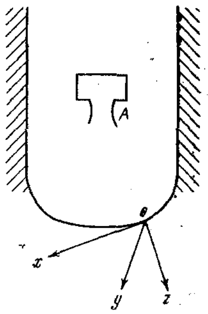
One of the promising methods of well-sinking, expecially in drilling through hard rock, involves the use of a high-temperature jet of gas. At the points of intensive heating the rocks fracture under the action of thermal stresses. Under optimum conditions the rock undergoes fragmentation into small pieces, and the rock material does not fuse.



We shall formulate and solve the high-temperature drilling problem on the assumption that the rock is elastic and fusion phenomena do not occur. Expressions will be obtained in a simple closed form for the rate of stationary drilling and the size of the particles in the fragmented material.

1. Formulation of the problem. Consider an infinite, uniform, and isotropic elastic body with an axially symmetric cavity in the form of a semi-infinite cylinder with a rounded base (see Fig. 1). A high-temperature jet of gas is played on the bottom of the cavity. The jet originates from a reservoir with nozzle A. Thermoelastic stresses appear in the medium in accordance with the Duhamel-Neumann law [1]. External stresses are assumed to be negligible in comparison with the characteristic temperature stresses. The surface region of the body undergoes fragmentation for sufficiently high stresses, and the resulting particles are carried off by a gas jet. Brittle fragmentation is assumed, and fusion effects are absent. These conditions impose a certain restriction on the temperature distribution in a purely brittle fragmentation.

We shall employ the following basic assumption:

$$\varkappa / vd \ll 1 \qquad (\varkappa = k/\rho c), \tag{1.1}$$

where $\varkappa$ is the thermal diffusivity, k is the thermal conductivity, ρ is the density, c is the specific heat, d is a typical linear dimension of the body (for example, the radius of curvature of the bottom, or the radius of the cylinder), and v is the normal drilling velocity, i. e., the rate of displacement of the boundary of the body (as a result of the removal of the fragmented material) along the normal to the surface.

The assumption given by Eq. (1.1) means that the external temperature field at each point on the boundary of the body penetrates to a depth which is small in comparison with the characteristic linear size of the part of the boundary which is subject to intensive heating. The condition given by Eq. (1.1) is satisfied by low values of thermal diffusivity for most rigid rocks. For example, if we suppose that $d \sim 10$ cm and take $\varkappa = 10^{-3} - 10^{-2}\,\mathrm{cm^2/sec}$, which is realistic for typical rocks, we find from Eq. (1.1) that $v \gg 10^{-4} - 10^{-3}$ cm/sec. This condition is not too restricting in view of the average rate of well-sinking.

From Eq. (1.1) the normal rate of drilling at a given point on the surface of the body is completely determined by the local flow parameters and the parameters of the body itself in the neighborhood of this point. We shall use the Cartesian coordinates xyz with the origin at a point 0 on the surface of the body, and the z axis normal to the surface in the inward direction. The stress, deformation, and temperature fields at 0 will then be slowly varying functions of x and y but rapidly varying functions of z. These fields form a peculiar wall layer. For the elastic displacement vector u, v, w, and the temperature T in the neighborhood of 0 we have from Eq. (1.1) in the usual approximation

$$u = v = 0, \quad w = w(z, t), \quad T = T(z, t). \tag{1.2}$$

If we neglect inertial forces we find that the Lame equation assumes the form [1]

$$(\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} - (3\lambda + 2\mu)\alpha\frac{\partial T}{\partial z} = 0, \tag{1.3}$$

where α is the linear expansion coefficient, and λ and μ are the Lame constants. The elastic and thermal constants of the body are assumed to be temperature-independent for the sake of simplicity. The initial temperature of the body is assumed to be zero throughout.

From the conditions at infinity

$$\partial w/dz = 0, \; T = 0 \quad \text{for} \quad z \to \infty, \tag{1.4}$$

and the Lame equation (1.3) we find that

$$\frac{\partial w}{\partial z} = \frac{3\lambda + 2\mu}{\lambda + 2\mu}\alpha T. \tag{1.5}$$

From Hooke's law and Eq. (1.5) we find that the principal stresses are given by

$$\sigma_x = \sigma_y = -\frac{\alpha E T}{1 - \nu}, \qquad \sigma_z = 0. \tag{1.6}$$

Therefore, near the surface we have a two-dimensional stress state with multilateral compression. The compressing stress is a maximum on the surface and decreases rapidly with increasing depth z. The temperature of the body satisfies the heat-conduction equation

$$\varkappa\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \qquad (z \geqslant 0) \quad (t - \text{time}), \tag{1.7}$$

and the boundary heat transfer condition at z = 0

$$-k\frac{\partial T}{\partial z} = h(T_0 - T), \tag{1.8}$$

where T_0 is the temperature of the incident gas at the point 0, and h is the heat-transfer coefficient.

It is important to note that when Eqs. (1.2)-(1.8) are valid for stationary fragmentation, they are even more valid under the corresponding nonstationary conditions.

2. Stationary drilling. 2.1. Let us first determine the normal rate of stationary drilling. Suppose that the boundary of the half-space is displaced with constant velocity v along the normal to its plane, and the gas temperature T_0 and the coefficient h are constant on the boundary. To find the temperature of the body we must solve the following boundary-value problem:

$$\begin{gathered}\varkappa\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \qquad (z - vt > 0),\\ -k\frac{\partial T}{\partial z} = h(T_0 - T) \qquad (z = vt),\\ T = 0 \quad \partial T/\partial z = 0 \quad \text{for} \quad z - vt \to \infty\end{gathered} \tag{2.1}$$

The solution of this problem is

$$T = \frac{h\varkappa T_0}{kv + h\varkappa}\exp\left[-\frac{v(z - vt)}{\varkappa}\right]. \tag{2.2}$$

The displacement of the boundary occurs as a result of continuous fragmentation of the body, and the removal of the fragments by the stream. The stress state of the body is described by Eq. (1.6). We shall assume that the following additional boundary condition is valid on the free surface of the half-space:

$$\sigma_x = \sigma_y = -\sigma_s \quad (z = vt), \tag{2.3}$$

where σ_s is the compression strength of the body.

We then find from Eqs. (1.6), (2.2), and (2.3) that the normal drilling velocity is

$$v = \frac{h\varkappa}{k}\left[\frac{\alpha E T_0}{(1-\nu)\sigma_s} - 1\right]. \tag{2.4}$$

From the solution of the problem given by Eqs. (2.2) and (2.4) we can readily find the condition for the absence of fusion effects:

$$\frac{(1-\nu)\sigma_s}{\alpha E T_n} < 1, \tag{2.5}$$

where T_n is the melting point.

2.2. We shall now estimate the size of the particles produced as a result of fragmentation. To do this we must consider in somewhat greater detail the fragmentation mechanisms operating in the surface layer. Two such mechanisms are possible in the case of compression. One of them is connected with the propagation of cracks [2], and the other with the loss of stability and local inhomogeneity of the material [2]. In our problem, the first mechanism operates during the initial stage of crack development while the second mechanism predominates during the final stage, since thermal stresses are concentrated in a narrow layer near the surface, and a thin plate of material in this layer is under the action of two equal principal compressing stresses. The particles of the fragmented material are therefore thin plates of thickness δ which is much less than the characteristic linear size b in the transverse direction.

To estimate δ we shall assume that the entire elastic energy of the particle which it has prior to fragmentation is converted into the effective surface energy of this particle as a result of fragmentation, which is given by

$$\delta S\, U = \gamma\,(2S + p\delta), \tag{2.6}$$

where U is the elastic energy per unit volume, γ is the effective surface energy per unit area, and S and p are the area and perimeter of the particle in plan, respectively.

According to Eq. (2.3)

$$U = \frac{1-\nu}{E}\,\sigma_s^2, \tag{2.7}$$

and if we suppose that $p\delta \ll 2S$, we find from Eq. (2.6) that

$$\delta = \frac{2\gamma E}{(1-\nu)\sigma_s^2}. \tag{2.8}$$

The quantity b evidently depends on the ratio of the compression strength σ_s and the tensile strength σ_p, since when the plate is pulled from the main body, one face of the plate experiences normal tensile stress

$$b = \delta f\,(\sigma_s/\sigma_p), \tag{2.9}$$

where f is a certain function.

Physical considerations suggest that when $b \gg \delta$ we must have $\sigma_s \gg \sigma_p$ and, conversely, when $\sigma_s \sim \sigma_p$ we should have $b \sim \delta$, i.e., the mechanism involving loss of stability no longer operates. It can be shown in the latter case that we can again use Eq. (2.8) to estimate δ from known specific surface energy γ.

Consider a numerical example. For silicate glass $(\pi E\gamma)^{1/2} \approx 5\cdot 10^3$ kg/cm$^{3/2}$, $\nu = 0.25$, $\sigma_s = 10^4$ kg/cm^2. Hence it follows from Eq. (2.8) that $\delta \approx 0.2$ cm. It is important to note that the values of γ for rocks have not been extensively investigated.

3. Nonstationary problem. 3.1. Let us return to the original axially symmetric drilling problem (Fig. 1). We shall consider stationary drilling, in which case the temperature of the body and the shape of the cavity depend only on the variables $\zeta = z_1 - v_* t$ and ρ, where z_1, ρ are cylindrical coordinates (ρ = 0 on the axis of symmetry), and v_* is the rate of drilling. In the present case the temperature and velocity of the gas jet and, consequently, the heat transfer coefficient are different at each point on the surface of the cavity, so that the normal drilling rate v at each point will be related to the unknown shape of the cavity $\zeta = \zeta(\rho)$ as follows:

$$v = v_* / \sqrt{1 + [\zeta'(\rho)]^2}. \tag{3.1}$$

At the point where the gas stream comes to rest, which lies on the axis of symmetry and on the surface of the body ($\zeta = 0$, $\rho = 0$), the normal drilling rate v is equal to the drilling rate v_*. Using Eq. (2.4) which gives the normal drilling rate, we obtain the following expression:

$$v_* = \frac{h_*}{\rho c}\left[\frac{\alpha E T_*}{(1-\nu)\sigma_s} - 1\right]. \tag{3.2}$$

Thus, the drilling rate v_* is completely determined by the following parameters: the heat-transfer coefficient h_* between the gas and the solid at the point where the jet comes to rest, the stagnation temperature T_* of the gas stream, the density of the body, its specific heat c, Young's modulus E, the linear thermal expansion α, the Poisson ratio ν, and the compression strength σ_s of the body.

The shape of the cavity in quasi-stationary approximation can be determined by solving the gasdynamic temperature problem. The additional boundary conditions for the temperature on the unknown contour are given by Eqs. (3.1) and (2.4). Imai's formula [3], obtained from the boundary-layer equations, can be used to determine the heat-transfer coefficient h. There is a device, however, which can be used to obtain an approximate solution of the cavity-shape problem. The temperature T_0 and the stream velocity U_0 on the surface of the body (on the outer boundary of the boundary layer) will be approximated by the functions

$$T_0 = \varphi(s), \quad U_0 = \psi(s) \quad (s \text{ — arc length}), \tag{3.3}$$

which are chosen on the basis of convenience and are specified to within a number of adjustable constants. If we solve the boundary-layer equations subject to Eq. (3.3) on the outer surface, and use the condition

$$T = \frac{(1-\nu)\sigma_s}{\alpha E}, \tag{3.4}$$

for the temperature of the gas on the surface of the body (found from Eqs. (1.6) and (2.3)), we find the heat-transfer coefficient

$$h = \omega(s), \tag{3.5}$$

which also depends on a number of constants.

If we eliminate v from (2.4) and (3.1) in the quasi-stationary approximation, we obtain the following relation connecting the shape of the cavity $\zeta = \zeta(\rho)$ with the heat transfer coefficient h and temperature T_0:

$$\frac{1}{\sqrt{1 + [\zeta'(\rho)]^2}} = \frac{h\,[\alpha E T_0 - (1-\nu)\sigma_s]}{h_*\,[\alpha E T_* - (1-\nu)\sigma_s]}. \tag{3.6}$$

In terms of the parametric variable s (length of arc), and using Eqs. (3.3) and (3.5), we obtain

$$\rho = \int_0^s \theta(s)\,ds,$$

$$\zeta = \int_0^s \sqrt{1 - \theta^2(s)}\,ds, \quad \theta(s) = \frac{\omega(s)\,[\alpha E\varphi(s) - (1-\nu)\sigma_s]}{\omega(0)\,[\alpha E\varphi(0) - (1-\nu)\sigma_s]}, \tag{3.7}$$

which is the required equation for the shape of the cavity in parametric form.

From the shape of the cavity (determined to within a number of constants) we can determine the gas flow in the cavity, and then find the undetermined constants. In this way, all three problems, i.e., the flow of the ideal gas in the cavity, the flow of a viscous gas in the boundary layer, and the fragmentation of the solid under the action of the temperature stresses are found to be closely related. The above method of solving the combined problem can also be used to obtain the exact solution, although this will, of course, require the use of computers.

3.2. Finally, we must determine the error which is introduced by replacing the nonstationary problem (due to the curvilinear shape of the cavity) by the quasi-stationary problem of solid fragmentation. To do this, we must estimate the characteristic time τ which is necessary to reach the stationary state. The condition for quasi-steadiness can then be written in the form

$$\tau v \ll d \ , \qquad (3.8)$$

where v is the normal drilling velocity and d is a typical linear dimension of the cavity. From Eqs. (1.7) and (1.8) we can determine τ by solving the boundary-value problem

$$\varkappa \frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \qquad [z - v(t)] \, t > 0) \qquad T = 0 \quad \text{for } t = 0;$$

$$-k \frac{\partial T}{\partial z} = h(T_0 - T) \quad \text{for } z = 0, \ 0 < t < t_1,$$

$$T = T_1, \ \frac{\partial T}{\partial z} = Q \qquad \text{for } z = v(t)\, t, \ ((v(t) > 0), \ t > t_1),$$

$$\left(T_1 = \frac{(1-\nu)\sigma_s}{\alpha E}, \ Q = -\frac{h}{k}(T_0 - T_1)\right). \qquad (3.9)$$

Dimensional analysis shows that

$$\tau = \beta T_1^2 / \varkappa Q^2 \quad \text{for} \quad t_1 \ll \tau \ , \qquad (3.10)$$

where β is a constant factor.

Using Eqs. (2.4) and (3.10), we can reduce Eq. (3.8) to the form

$$\beta \varkappa / vd \ll 1 \ . \qquad (3.11)$$

As can be seen, this quasi-steadiness condition follows from the basic assumption given by Eq. (1.1).

REFERENCES

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